A new dynamic methodology for stationary and transient mass flow rate measurements

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Résumé :

Cet article rapporte l'analyse expérimentale et numérique des flux de gaz raréfiés dépendants du temps à travers un micro-tube métallique long. La méthodologie expérimentale a été concue sur la base de la technique du volume constant et adaptée pour mesurer l'évolution avec le temps d'un débit massique transitoire à travers un micro-tube. En outre, le temps caractéristique de chaque expérience, extrait des mesures de pression dans chaque réservoir, a offert une indication claire sur la dynamique du flux transitoire en fonction de son niveau de raréfaction. De plus, nous présentons une méthodologie originale pour extraire les débits massiques stationnaires en utilisant la conductance du tube, qui peut etre associée au temps caractéristique de l'expérience, mesurée pour différentes pressions moyennes entre deux réservoirs. La conductance mesurée du tube et le débit massique stationnaire à l'entrée et à la sortie du micro-tube a été comparé aux résultats numériques obtenus avec le modèle d'équation cinétique linéaire BGK. Les résultats ont été obtenus dans un large éventail de conditions de raréfaction pour l'azote (N_2)

Abstract :

This paper reports the experimental and numerical analysis of time-dependent rarefied gas flows through a long metallic micro-tube. The experimental methodology was conceived on the basis of the constant volume technique and adapted to measure the evolution with time of a transient mass flow rate through a micro-tube. Furthermore, the characteristic time of each experiment, extracted from the pressure measurements in each reservoir, offered a clear indication on the dynamics of the transient flow as a function of its rarefaction level. Moreover we present an original methodology to extract stationary mass flow rates by using the tube conductance, which can be associated to the characteristic time of the experiment, measured for different mean pressures between two tanks. The measured conductance of the tube and the stationary mass flow rate at the inlet and outlet of the micro-tube was compared to numerical results obtained with the BGK linearized kinetic equation model. The results were obtained in a wide range of rarefaction conditions for nitrogen (N_2).

Mots clefs : Rarefied Gas, Transient Flow, Mass flow Rate Measurement

1 Introduction

In gas micro-fluidic-devices the equivalent mean free path of the gas molecules (ℓ) can be of the same order as the characteristic dimension (L) of these devices. In this case the fluid can be considered to be under rarefied conditions and it cannot be treated as a continuum medium as it is usually done by classic fluid mechanics. Depending on their level of rarefaction,which can be characterized by a rarefaction parameter $\delta = L/\ell$, micro gas flows are often modeled by means of molecular based approaches. In practical applications, these microfluidic devices are often required to function in transient conditions, hence a time-dependent analysis of the flow is needed. To this day not much attention has been dedicated to study and analyze time-dependent gas flows in micro devices, to the extent that experimental data on the matter is basically lacking.

Gas rarefaction induces macroscopic non-equilibrium effects at the interface between gas and solid surfaces for the gas macroscopic parameters, such as velocity slip and temperature jump at the wall. These non-equilibrium phenomena are affected by the specific configuration of the surface of the micro-device, such as surface roughness and materials, and the molecular structure of the gas considered.

The main focus of attention in rarefied gas flows has been the measurement of stationary flow configurations in order to obtain accommodation coefficients. These empirical coefficients can be as important as the transport coefficients and are to be introduced in modified boundary conditions of the Navier-Stokes equations. However, recently, some authors have started paying attention to transient rarefied gas flows, too, by numerically investigating the phenomenon ([2, 6, 12, 9]). Nevertheless, very little experimental efforts on the topic have been conducted so far.

Due to this lack of experimental observation, the primary goal of the present study was to measure and analyze the relaxation process of a gas diffusing through a long micro-tube. The relaxation process refers to the pressure variation with time in two tanks, set at the inlet and outlet of a microtube, from an initial pressure difference stage until a final equilibrium stage of pressure equality. The study has been conducted for arbitrary rarefaction conditions that range from near hydrodynamic to transition regime. Furthermore, by monitoring the entire relaxation process of pressure evolution inside two tanks of equal volumes at the inlet and outlet of the micro-device, we demonstrated that it is indeed possible to extract an unique conductance value that depends only on the average pressure of the experiment, the geometry of the channel and the gas nature. This was achieved by means of a dynamic constant volume technique that was firstly proposed to measure thermally driven gas flows by [8]. The originality in respect to the classic constant volume technique ([1], [13], [3]) relies on the fact that a dynamic measurement technique considers the time-dependency of pressure during the full duration of the pressure relaxation process obtained from one single experiment. The methodology therefore takes under account the intrinsic non-stationarity of the pressure measurements and profits from it to characterize the relaxation process by a characteristic time that can be associated to the conductance of the channel used.

From these experiments it is not only possible to measure a single conductance value, but it is also possible to associate to the pressure evolution with time the time-dependent mass flow rate along the micro-tube.

The experiments were performed for different initial pressure ratios and for a large spectrum of rarefaction conditions.

Finally, the obtained experimental results were compared with the numerical solution of the linearized BGK model kinetic equation in the case of stationary flows ([4]).

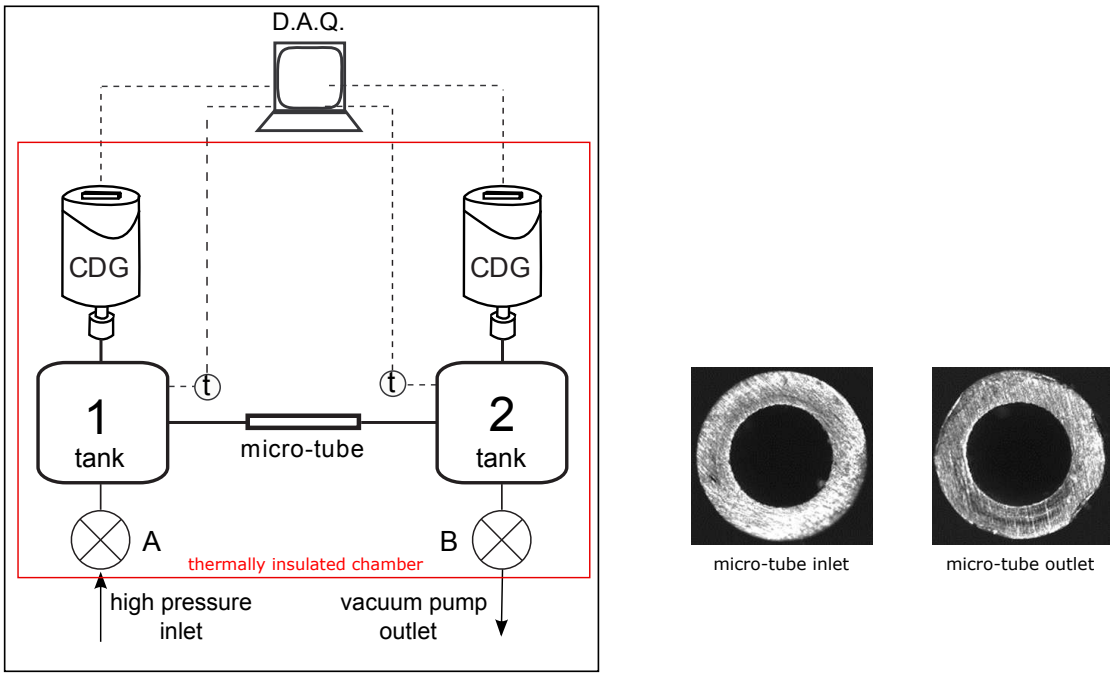

FIGURE 1 – The experimental apparatus composed by the micro-tube, two temperature sensors ($t1$ and $t2$), two tanks (R1 and R2) and the capacitance pressure diaphragm gauges (CDG). And details of the circular cross-section micro-tube.

The understanding of the phenomena relating the non-equilibrium effects of viscous slip to the transient gas macroscopic displacement in micro-systems under different type of rarefaction conditions and for different gases could be used to develop interesting applications such as for example gas separators or accurate micro mass flow rates regulating devices.

2 Experimental Apparatus

The experimental set-up was composed of a single metallic (stainless-steel) micro-tube of circular cross-section, two reservoirs, two capacitance diaphragm pressure gauges, two thermocouples, a vacuum pump and the acquisition system (Figure 1). The single metallic micro-tube ($L_t = 92.22 \pm 0.01mm$, $D = 435.5 \pm 3.5\mu m$) was connected to two tanks which were positioned at the inlet and outlet of the capillary. The volume of the tanks was chosen to be much larger in respect to the volume of the micro-tube. The main experimental campaign was performed for a set of volumes with almost equal dimensions, that is $V_1 = 173.2 \pm 0.5\, ml$ and $V_2 = 174.5 \pm 0.5\, ml$ ($V_1/V_2 = 0.9926$). The indexes 1 and 2 stand for inlet and outlet of the tube, respectively. The two tanks were appositely designed to host the single micro-tube. However, the experimental apparatus can be re-arranged in order to host in its test-section a great variety of micro-fluidic devices.

The temperature inside both tanks was continuously monitored by means of two thermocouples and it was stabilized at around $T_m = 295.5 \pm 0.5K$ during the full duration of one experiment. The apparatus was thermally insulated from the external room by means of the same adiabatic chamber used by [7]. Additionally, an efficient isothermal stability was achieved due to the high thermal inertia of the two stainless steel tanks. The temperature oscillations at the inlet and outlet tanks were evidently lower than the sensibility of the temperature sensors. We estimated these oscillations in the order of $dT/T_m = 10^{-3}$.

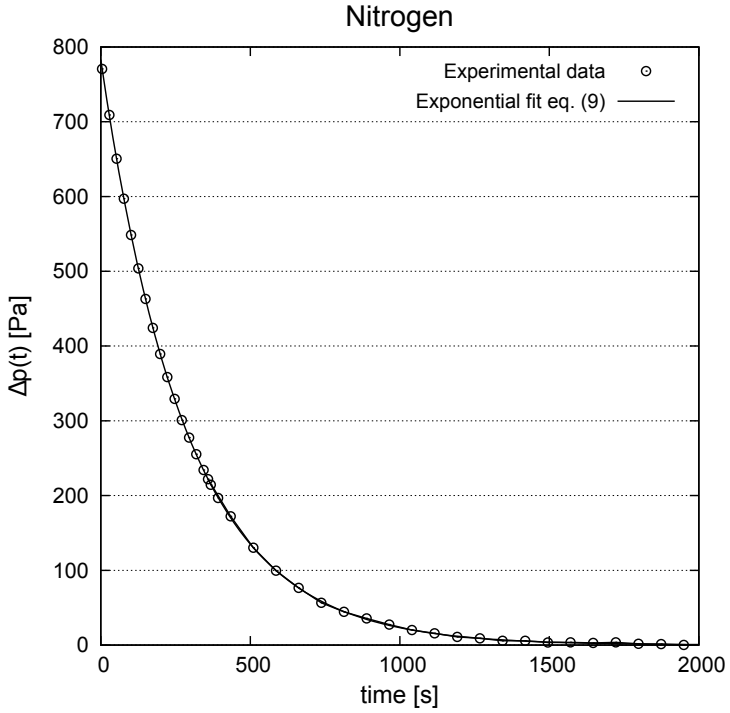

FIGURE 2 – Exponential pressure difference variation with time $\Delta p(t)$ and its fitting function.

The inlet tank was connected by means of valve A to a high pressure reservoirs containing nitrogen N_2. The outlet tank was connected by means of valve B to a vacuum pump. The test section could be vacuumed until pressures as low as $10^{-2}Pa$ and it was regulated by means of valve B. The rarefaction conditions imposed in the micro-tube ranged from transition to near hydrodynamic regime.
The fast response capacitance diaphragm gauges (CDG) monitored the pressure variation with time inside the two tanks. The acquisition frequency of the pressure gauges was of $33Hz$. CDGs with different full scales were used as a function of the nominal pressure in the system, which allowed us to improve the measurements accuracy at low pressures.

3 Experimental methodology

The hereafter proposed technique overcomes some of the main limitations of the classical constant volume technique for mass flow rate measurements. The main limit of the constant volume technique results from the fact that the method does not provide direct access to a real transient gas flow. In fact, the method could only be used at a fixed time during one experiment, allowing us to derive various stationary mass flow-rates related to various quasi-stationary states.
Now considering precisely the use of the methodology at a fixed time, some limits can be found considering the two opposite purposes of the technique. A time interval dt was to be defined at the instant where the pressure measurement had to be made. This time interval had to be sufficiently small to allow the identification of a quasi-stationary state, in other words to a mass flow rate considered as stationary. But, nevertheless, it was necessary to consider a time interval dt, sufficiently long, that allowed the extraction of a pressure variation measurement that had a substantial physical meaning.
Thus, in some cases, where a very fast pressure variation with time was initially induced, it might have been difficult to completely verify both the previously defined constraints : in such cases the error on the

stationary mass flow rate evaluation would be without any doubt increased. When the pressure varies with time in a linear way, this error becomes, by all matters, completely negligible.
The technique proposed herein is not limited to a quasi-stationary approach, *i.e.* to the measurement of a linear pressure variation with time, as commonly done in the literature by using the constant volume technique methodology [1, 3, 11].
The herafter introduced technique has been developed initially to measure thermally driven gas flows by [8]. This dynamic method allows us to obtain, through the pressure variation with time time-derivative, an explicit expression of the transient mass flow rate flowing through the micro-device. The new methodology eliminates the necessity of using a time interval, where the pressure varies linearly, but uses the whole process of the pressure decay in both reservoirs.
Additionally, this methodology offers new interesting possibilities, since the pressure exponential decay with time between both reservoirs can be associated to a conductance value which depends only on the tube geometry, the gas used and the average pressure in the micro-device. By analogy to Ohm's law one can relate the pressure difference $(p_1 - p_2)$ across the micro-tube to the "potential", the flow throughput $\dot{Q}$ to the "current" and the conductance C to the inverse of the "an electrical resistance". Therefore, one can define the conductance of the tube as

$$C = \frac{\dot{Q}_2}{p_1 - p_2}, \tag{1}$$

The flow throughput $\dot{Q}$,commonly used in the field of vacuum science and technology to express gas flow rates [5], is defined as

$$\dot{Q}_i = \frac{d(pV)_i}{dt} = RT\dot{M}_i, \tag{2}$$

and it can be easily correlated to the mass flow rate entering the reservoir by using the ideal gas equation of state eq.. From hereon, one can proof that the conductance C can be related to the pressure difference variation in time of the system in the following form :

$$\Delta p(t) = \Delta p_0 \, exp\left(-\frac{t}{\tau}\right), \tau = \frac{V_0}{C}, \tag{3}$$

where Δp_0 is the initial pressure difference at $t = 0$ and τ is the characteristic time of the experiment or the system relaxation time.
The pressure difference variation with time between tank 1 and 2 can be thus associated to an exponential decay for the case where the conductance of the tube is constant during one single experiment. In addition we can note that τ can be extracted from the experimental pressure variation with time (Figure 2). This value of the characteristic time allows us to obtain the tube conductance corresponding to the mean pressure of an experiment. Finally, from equations 2 and 3, the pressure difference decay can be associated to the transient mass flow rate diffusing through the tube as

$$\dot{M}(t) = \frac{V_0}{RT}\frac{\Delta p_0}{\tau} exp\left(-\frac{t}{\tau}\right). \tag{4}$$

4 Results

The main result of this study is to be attributed to the methodology itself, which is able to associate to the pressure variation with time, induced by gas diffusing through a micro-tube, to the extraction of a

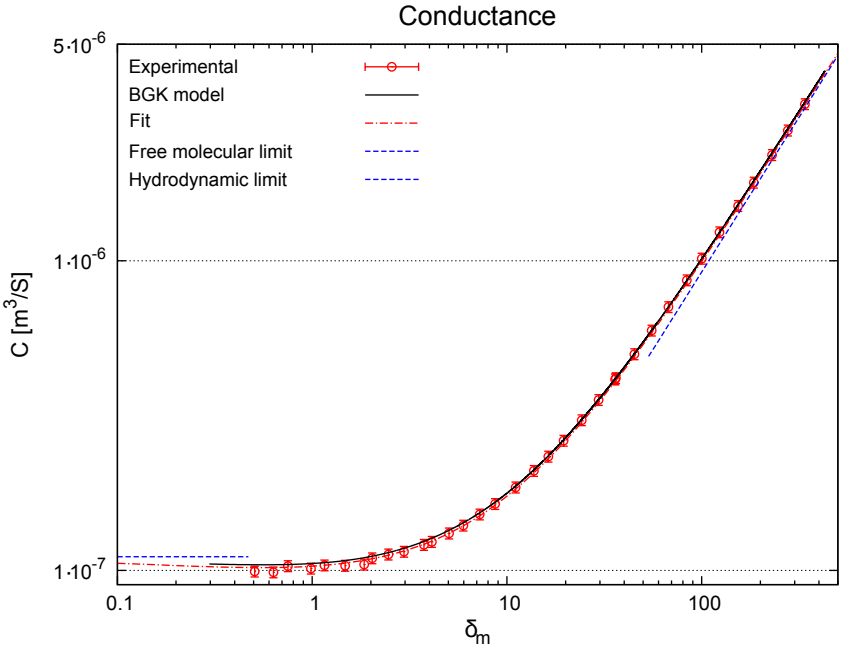

FIGURE 3 – Left : Exponential pressure difference variation with time $\Delta p(t)$.Right : Conductance ($C = V_0/\tau$) for nitrogen (N_2) as a function of the mean gas rarefaction conditions, that is from transition to near hydrodynamic regime.

conductance value at arbitrary rarefaction conditions. This is achieved by fitting the pressure variation with time, at different mean pressures, by means of eq. (3). For the same tube and volumes of the tanks, we observed, as expected, that the conductance was strongly influenced by the rarefaction conditions of the flow.

4.1 Conductance and stationary mass flow rate

The first step for the measurement of transient mass flow rates is the measurement of the conductance values of the tube for a given gas at a given mean pressure. The conductance can be easily associated to the characteristic time of the relaxation process (eq. 3). We compared our conductance experimental results to numerical results of the dimensionless mass flow rate G, obtained by [4] with the BGK model kinetic equation and to the empirical fit of G, obtained by [10], that reproduces very accurately the BGK solution. For the case shown on Figure 3, the mean relative deviation between experimental and numerical results is of the order of 1.3%.
From the non stationary results obtained it is also possible to extract stationary mass flow rate results by means of the conductance results. Therefore, we here show results of stationary mass flow rates obtained by using the dynamic constant volume technique and we compare them to results obtained numerically from the BGK model kinetic equation ([4]).

Let us remember that the conductance, for a fixed tube geometry, varies only as a function of the mean rarefaction (or mean pressure) and of the gas used. This means that the conductance does not depend on the initial pressure difference imposed.
Therefore, in order to extract a stationary mass flow rate, once the conductance has been measured for a

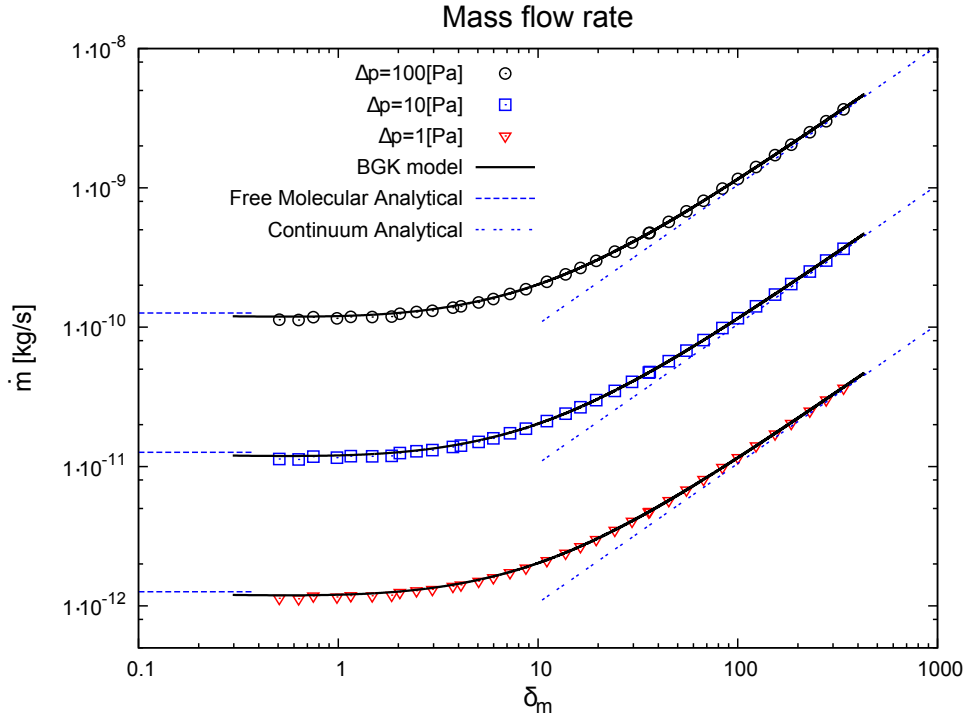

FIGURE 4 – Comparison between numerical and experimental results of stationary mass flow rates as a function of rarefaction for several pressure differences imposed for nitrogen. The uncertainties associated to the measurement are smaller than the size of the points ($\Delta\dot{M}/\dot{M} = 3\%$).

given rarefaction condition, one can reduce eq. (4) in the limit of $t \to 0$ to

$$\dot{M} = \frac{C}{RT}\,\Delta p_0, \qquad C = f(\delta_m, gas). \tag{5}$$

If one knows the conductance for one precise rarefaction condition, one can obtain the mass flow rate along the tube for an arbitrary difference of pressure imposed.

This property which is characteristic of the conductance parameter is very useful, since normally mass flow rates engendered by very low pressure differences are extremely difficult to measure. Nevertheless, with this technique one could apparently measure mass flow rates at arbitrary pressure differences imposed. The here presented mass flow rate measurements have been calculated for $\Delta p = 100Pa$, $\Delta p = 10Pa$ and $\Delta p = 1Pa$, but other pressure differences could have been used.

4.2 Mass flow rate variation with time

By fitting the data for pressure variation with time results by means of the exponential function of eq. (3) and consequently by using eq. (4), the dynamic constant volume technique allowed us to measure non stationary mass flow rates along the full duration of one experiment, that is from the initial pressure difference imposed, where the mass flow rate along the tube was at its maximum, until a final pressure equality equilibrium stage was reached, where the mass flow rate was zero.

It is possible to notice that the influence of the rarefaction on the flow is predominant in the mass flow rate evolution when different rarefaction conditions are compared whereas keeping the same gas and same initial pressure difference imposed (Fig. 5). At rarefaction conditions tending to hydrodynamic flow regime, for example at $\delta_m = 338$, the initial pressure difference imposed engendered a greater ini-

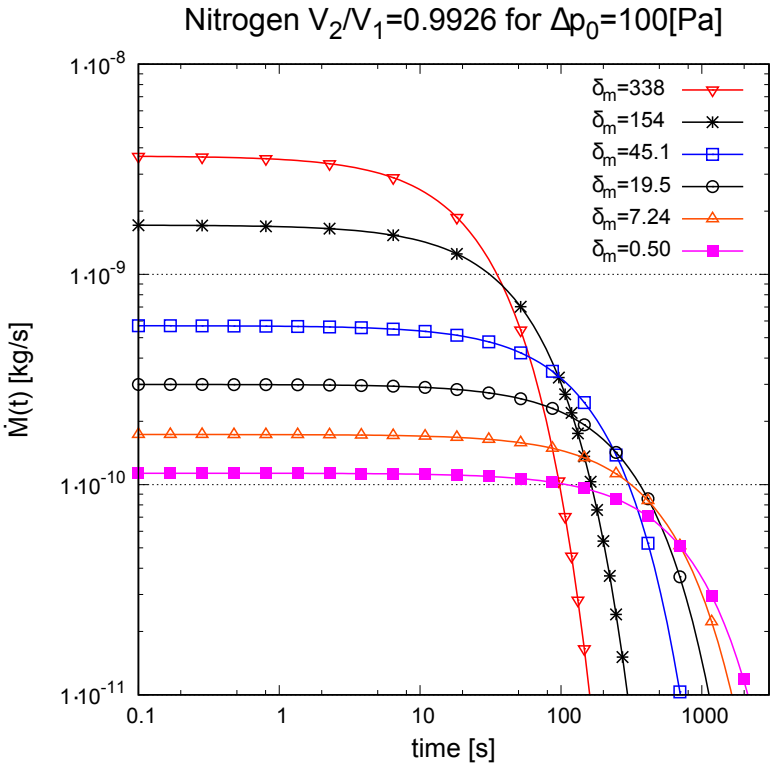

FIGURE 5 – Mass flow rate variation with time until a final equilibrium of no motion is reached through the tube for the same initial pressure difference.

tial mass flow rate, and its final equilibrium state to be reached more rapidly its final equilibrium stage in respect to a higher rarefied flow, for example at $\delta_m = 0.50$ in transition regime.

5 Conclusions

This work is a first effort to experimentally analyze transient rarefied gas flows through long tubes. This study is aimed to add knowledge on time-dependent micro gas flows which is at the present moment lacking in the literature. Specific applications of transient micro flows could be applied to gas separators, micro chromatography, oscillating micro actuators and others.

Of novelty and particular importance is the new methodology proposed in order to extract from a theoretically derived exponential expression characteristic times of single experiments which can characterize the relaxation phenomenon of pressure variation with time as a function of gas molecular weight and gas rarefaction.

The here proposed study has been conducted for fixed parameters such as the diameter and length of the tube. The mass flow rate variation with time behavior as a function of the gas rarefaction and molecular weight was analyzed.

Moreover, the dynamic constant volume technique offers the possibility to extract stationary values of mass flow rate by means of the conductance C, which was obtained from the time-dependent experiments. It was possible to obtain a large spectrum of results from a reduced number of experiments performed. This final result of the paper represents a significant accomplishment.

Finally, the experimental stationary mass flow rates results were compared to results obtained from the

BGK kinetic model equations and the agreement is excellent for flows in slip and transitional regime.

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